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Scaling behavior of velocity and temperature in a shell model for thermal convective turbulence

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Based on the conservation of total energy and entropy under conditions of nonviscosity and forcing-free, a shell model is developed to simulate the thermal convective turbulence. A fluxlike coupling mode is supposed to ensure that the total exchanging energy (EE) between the velocity and temperature is conserved when omitting the buoyancy, external forcing, and viscous effects. We call this EE conservation. Under such assumptions, the relative scaling exponents of structure functions with order p are almost parameter independent and consistent with a multiplicative cascade model presented by She and Leveque [Phys. Rev. Lett. **72**, 336 (1994)]. Moreover, the energy spectra with different controlling parameters can be collapsed into a single curve by Kolmogorov scaling. Otherwise, the model behaviors may be parameter dependent.
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I. INTRODUCTION

It is generally believed that fully developed turbulence has an inertial range in which the fluid motions show universal behaviors. In particular, the structure functions may depend on the eddy scale by a power law, i.e., $\langle \delta v_l^p \rangle \sim l^{\zeta_p}$ (where δv_l is longitual velocity difference at distance l, p is an arbitrary real number, and $\langle \; \rangle$ is statistical average) and ζ_p is a scaling exponent. In recent years, much numerical and experimental evidence [1] show that ζ_p is a nonlinear function of p. This is supposed to be induced by turbulent intermittence. A variety of phenomenological models have been proposed by various authors to explain the intermittence mechanism. Among them the most recent and more plausible one is that of She and Leveque [2] (denoted as SL [2] hereafter). The model predicts that $\zeta_p = p/9 + 2[1-(\frac{2}{3})^{p/3}]$, which is supported by experimental results [3] and observations of magnetohydrodynamics turbulence [4]. Intuitively, if a scalar property T such as temperature and density is convected by the current field, we may expect that a similar scaling exists for the scalar structure functions, that is $\langle \delta T_l^p \rangle \sim l^{\xi_p}$. However, in the case of thermal convective turbulence, the picture is more complicated by the fact that the SL [2] scaling, and other scaling models [5], seemingly contradict the existing facts [6].

In the present paper, a shell model (or the Gledzer-Ohkitani-Yamada model) of thermal convective turbulence with neutral stratification is developed to study the scaling behaviors of velocity and passive scalars. Similar models have been presented by Jensen and co-workers [7], in which the scalar field is purely passively driven by currents, and by Brandenburg [8] and Suzuki and Toh [9] for stable stratification. Because the two latter models use real variables at each shell, they may not simulate the phase transfer of turbulence. Generally, a shell model replaces the global correlation of Navier-Stokes equations by localized nonlinear interactions while preserving some basic symmetries and conservation laws. In other words, the differences among these models are the choices of local interaction modes that determine the unstable fixed points and accordingly different dynamical behaviors may occur [10]. Kadanoff *et al.* [11] argued that by conserving kinetic energy and a helicitylike quantity $L = \Sigma(\varepsilon-1)^{-n}|u_n|^2$, the system behaves nearly universally. By constructing a model with only one conservation quantity, Gat, Procaccia, and Zeitak [12] found that $\zeta_p = p/3$. Therefore, we will restrict ourselves to searching for the relationship of conservation laws and dynamical behaviors and the conditions under which a shell model for thermal convective turbulence may seemingly have universality.

For thermal convective turbulence, the heat transport plays a key role in the motions. In some specific cases such as Rayleigh-Bénard convection, the total exchanging energy (EE), heat flux converted from external heating into the fluid motions, can be regarded as constant under certain conditions. Therefore, in our model the nonlinear coupling modes between shells are designed to conserve the EE in the limit of zero viscosity, forcing, and buoyancy. We shall call this EE conservation.

The paper will be organized as follows. Section II presents the construction of the model. Section III gives the scaling behavior of the structure functions. It is found that the relative scaling exponents, which are the powers of structure functions related to the third-order structure function within the inertial range, are almost parameter independent. For the sake of comparison, we apply the coupling modes suggested by Jensen and co-workers [7] to this problem and find that the scaling laws strongly depend on the controlling parameters. The normalized spectra and the corresponding energy communication scenario are discussed in Sec. IV. When EE is conserved, the spectra can be collapsed into a single curve by Kolmogorov scaling even in the dissipation region. The dynamical implication of the model and concluding remarks are presented in Sec. V.

II. MODEL

For thermal convective turbulence, in addition to some general symmetries, there are two important conservative

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quantities, i.e., the total energy E and the total entropy V in the limit of zero viscosity and forcing. In the context of theoretical and experimental studies, a horizontally periodic or homogeneous geometry is usually supposed, so that total vertical heat flux is independent of the height of the fluid layer [13]. In a steady state, a constant vertical heat flux in the atmospheric boundary layer can also be assumed to be a good approximation under the prerequisite of horizontal homogeneity [14]. Therefore, we hope that EE can be conserved also under certain conditions while keeping the conservation of E and V in our model. To describe the phase transfer, complex variables will be adopted. A Boussinesq approximation [15] is always supposed. With these in mind, the model can be constructed as

$$\frac{du_n}{dt} = -\nu k_n^2 u_n + ik_n\left(u_{n+1}^* u_{n+2}^* - \frac{\varepsilon}{2} u_{n+1}^* u_{n-1}^* - \frac{1-\varepsilon}{4} u_{n-1}^* u_{n-2}^*\right) - \tilde{\alpha}\theta_n,$$

$$\frac{d\theta_n}{dt} = -\kappa k_n^2 \theta_n + ik_n(\alpha_1 u_{n+1}^* \theta_{n+2}^* + \alpha_2 u_{n+2}^* \theta_{n+1}^* + \beta_1 u_{n-1}^* \theta_{n+1}^* - \beta_2 u_{n+1}^* \theta_{n-1}^* + \gamma_1 u_{n-1}^* \theta_{n-2}^* + \gamma_2 u_{n-2}^* \theta_{n-1}^*) + f_n,$$

where complex variables u_n and θ_n ($n=0,1,\ldots,N$) are velocity and temperature components for wave number k_n, and ν and κ are viscous and diffusive coefficients, respectively, throughout this study we choose $\nu=\kappa$ (Prandtl number Pr $=1$); $\tilde{\alpha}$ denotes the thermal convection of the velocity field, which is proportional to the Rayleigh number Ra; f_n denotes the external forcing, $k_n = k_0 2^n$, $i=\sqrt{-1}$, and the asterisks represents a complex conjugate operation. Obviously, the model herein meets the Liouville theorem $\Sigma_n \partial \dot{u}_n / u_n = 0$. The total energy E, entropy V, and exchanging energy H, can be defined as

$$E = \sum_{n=0}^{N} \left(|u_n|^2/2 + \tilde{\alpha}\int_0^t \mathrm{Re}(u_n^* \vartheta_n) dt \right),$$

$V=\Sigma_{n=0}^N |\theta_n|^2/2$, and $H=\Sigma_{n=0}^N \mathrm{Re}(u_n\theta_n^*)$, respectively. The coupling modes of the first equation guarantee that E is conserved in the nonviscous limit [note that $\Sigma\ \mathrm{Im}(u_n\theta_n^*)\equiv 0$ under this condition]. We will set $\varepsilon=\frac{1}{2}$, which means that the coupling of the velocity is based on the usual choice. Such a configuration implies that the velocity field obeys a nearly universal scaling [13] that seems closest to real turbulence. The other coefficients $\alpha_1,\alpha_2,\beta_1,\beta_2,\gamma_1,\gamma_2$ are to be determined. To keep V and H conserved, the nonlinear terms must have a fluxlike form. By denoting α_1 as τ, it is easy to find that

$$\gamma_1 = -\tau, \quad \alpha_2 = -\beta_1 = 1-\tau, \quad \gamma_2 = \beta_2 = \tau - \tfrac{1}{2}. \tag{1}$$

We call this model 1. After some manipulations we derive

$$\frac{d}{dt}|u_n|^2 \Big/ 2 = -\nu k_n^2 |u_n|^2 - \tilde{\alpha}\ \mathrm{Re}(u_n\theta_n^*) + F_u^n - F_u^{n-1},$$

$$\frac{d}{dt}|\theta_n|^2 \Big/ 2 = f_n\theta_n^* - \kappa k_n^2 |\theta_n|^2 + F_\theta^n - F_\theta^{n-1},$$

$$\frac{d}{dt}\mathrm{Re}(u_n\theta_n^*) = \mathrm{Re}[f_n^* u_n - (\nu+\kappa)k_n^2 u_n\theta_n^*] - \tilde{\alpha}|\theta_n|^2 + F_{u\theta}^n - F_{u\theta}^{n-1},$$

where F_u^n, F_θ^n, and $F_{u\theta}^n$ represent the corresponding fluxes transferring toward high wave numbers. Then the total entropy is conserved without forcing and viscosity, and it will be conserved statistically in the real model after some transient processes, i.e., $\langle V\rangle = \mathrm{const}$. However, the conservation of total exchanging energy requires no buoyancy. As a matter of fact, the exchanging energy at each shell decreases at a constant rate $\tilde{\alpha}\Sigma|\theta_n|^2$. Here we see that the buoyancy plays an important role in the cascading processes. While preserving total entropy cascading, the existence of buoyancy alters the energy transferring scenario of the velocity field. At each shell, even in the so-called inertial range, exchanging energy is supplied from the scalar field so that kinetic energy flux is not shell independent. Similarly, the EE may not obey a cascading scenario strictly. Nevertheless, our numerical results show that energy cascading persists and an obvious inertial region exists [see Fig. 3(a) below]. Moreover, the scaling rules of structure functions are almost independent of $\tilde{\alpha}$.

If we apply the coupling modes of Jensen and co-workers [7] to the present problem, the parameters will be

$$\alpha_1 = \alpha_2 = 1, \quad \beta_1 = \beta_2 = \tfrac{1}{2}, \quad \gamma_1 = \gamma_2 = -\tfrac{1}{4}. \tag{2}$$

Now $\tilde{\alpha}$ is a unique parameter that determines the coupling of the velocity and scalar field. The conservative quantities are the total entropy and energy. We shall refer to this as model 2 hereafter.

Before discretizing the equations, we introduce an artificial damping of the forms $-\nu' u_n/k_n$ and $-\kappa'\theta_n/k_n$ at small wave numbers since there is occasionally an inverse energy cascade in the model [9,16]. A second-order slave-frog Adams-Bashforth method [17] is employed. We choose $k_0=1$, $\nu=10^{-7}$–10^{-8}, $\Delta t=10^{-4}$, $\nu'=\kappa'=10^{-6}$, $f_3=(1+i)\times 10^{-3}$, $f_4=(1+i/2)\times 10^{-3}$, $f_n=0$ ($n\neq 3,4$), and the shell number $N=22$. All statistical quantities are calculated by averaging the ensemble 10×10^6 times after transient processes.

III. STRUCTURE FUNCTIONS

In the present model, the analogs of the velocity and temperature structure functions are $S_p=\langle|u_n|^p\rangle$ and $T_p=\langle|\theta_n|^p\rangle$. We find a power dependence $S_p\sim k_n^{-s_p}$, $T_p\sim k_n^{-s_p}$ in the range of $4\leq n\leq 13$, which is subject to slight variations with parameters. These functions have no period-three oscillation except when $\tau=0.5$, in which a triple product $(\langle|u_{n-1}|\rangle\langle|u_n|\rangle\langle|u_{n+1}|\rangle)^{1/3}$ is used to estimate S_p for

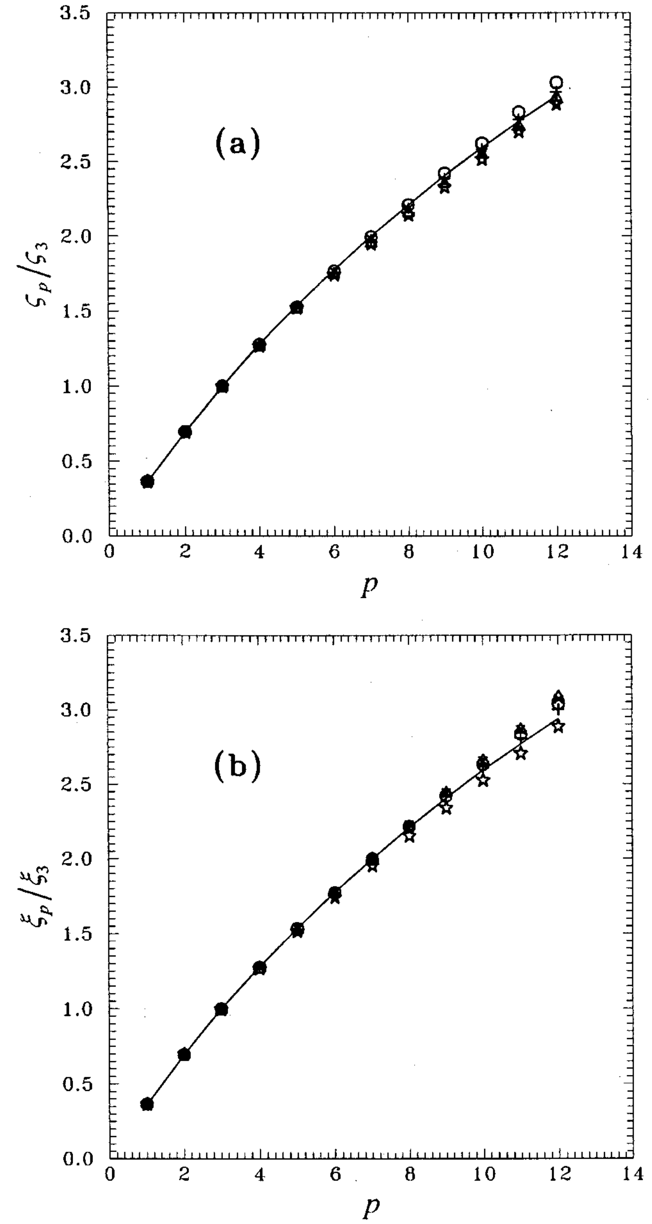

FIG. 1. Relative scaling exponents for various parameters: (a) ζ_p/ζ_3 and (b) ξ_p/ξ_3. (Circles, $\nu=10^{-8}$, $\tau=0.5$, and $\tilde{\alpha}=0.1$; triangles, $\nu=10^{-7}$, $\tau=0.5$, and $\tilde{\alpha}=0.1$; stars, $\nu=10^{-7}$, $\tau=0.7$, and $\tilde{\alpha}=0.1$; pluses, $\nu=10^{-7}$, $\tau=0.5$, and $\tilde{\alpha}=2$; crosses, $\nu=10^{-7}$, $\tau=0.5$, and $\tilde{\alpha}=10$; solid line, SL [2] scaling.)

eliminating the oscillation. T_p can be treated similarly. The exponents ζ_p and ξ_p can be determined accurately by measuring the slope of the log-log plot in the inertial range. As mentioned above, we find that ζ_p and ξ_p depend on parameters $\tilde{\alpha}$ and τ. By computing these indices at seven different $\tilde{\alpha}$ within the interval (0.001,10), we obtain the average of ς_3 and ξ_3 as 1.088 ± 0.045 and 1.163 ± 0.049, respectively. The system seems to be more stable to the variations of τ. The average of the exponents at $\tau=m/10$ $(m=1,2,\ldots,9)$ gives $\zeta_3=1.115\pm0.025$ and $\xi_3=1.163\pm0.015$. In a shell model with hyperviscosity, Leveque and She [18] found a similar dependence. Nevertheless, the relative scaling exponents ζ_p/ς_3 are parameter independent and obey the SL [2]

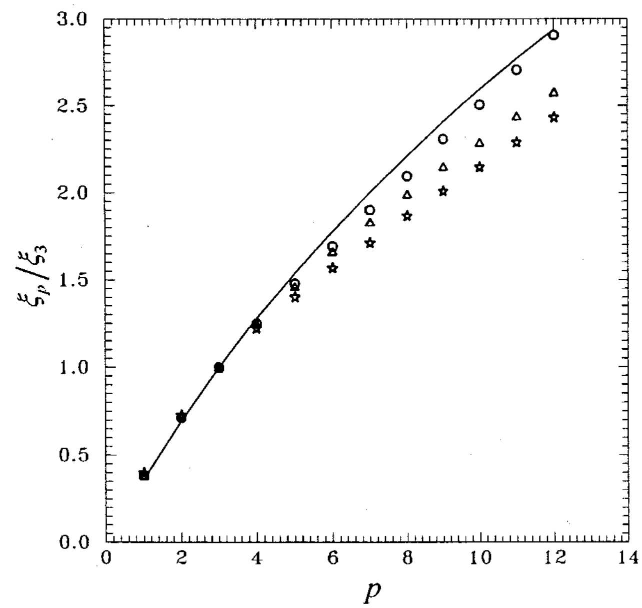

FIG. 2. Relative scaling ξ_p/ξ_3 by model 2. Circles, $\nu=10^{-8}$ and $\tilde{\alpha}=0.1$; triangles, $\nu=10^{-8}$ and $\tilde{\alpha}=0.5$; stars, $\nu=10^{-8}$ and $\tilde{\alpha}=2$.

scaling rule. One can compute ζ_p/ς_3 directly by determining ζ_p first. However, by plotting S_p versus S_3, the power scaling may extend to a wider range. This is a property recently discovered by Benzi *et al.* [3] and called extended self-similarity (ESS), which facilitates the determination of the relative exponents. In addition, such an operation can effectively reduce the oscillation of structure functions. ESS has been supported by many observations and numerical works [19]. So all the relative scaling exponents here are computed based on ESS. Under our assumption of constant energy communication, we find similar universality; specifically, not only the relative scaling exponents of the velocity structure function but also those of temperature obey the same SL [2] scaling and are almost parameter independent (Fig. 1).

When using model 2; similar universality for the relative scaling exponents of the velocity structure functions is observed, which means that the variation of nonlinear coupling in the scalar equation does not react to affect the velocity scaling behaviors. However, the relative scaling exponents of the temperature structure functions rely strongly on $\tilde{\alpha}$ (Fig. 2). The universality breaks down. We have tried many other coupling modes and all of them give a different ξ_p/ξ_3 for different coupling coefficients.

The agreement of the present model with SL [2] suggests a positive-energy cascading. In fact, the energy flux cascading from large scales to small scales is much larger than the exchanging energy between each shell. This indicates that nonlinear terms dominate in the energy balance and consequently the buoyancy does not change the cascading of kinetic energy much. From a dimensional view, $\langle F_u^n\rangle \sim k_n\langle u_n^3\rangle \sim k_n S_3$, so that a constant cascading flux of kinetic energy implies $\zeta_3=1$. However, a constant flow of total energy toward the high-wave-number end is necessary to sustain a positive cascading. Since the exchanging energy decreases with the scales, the kinetic-energy flux cascading to

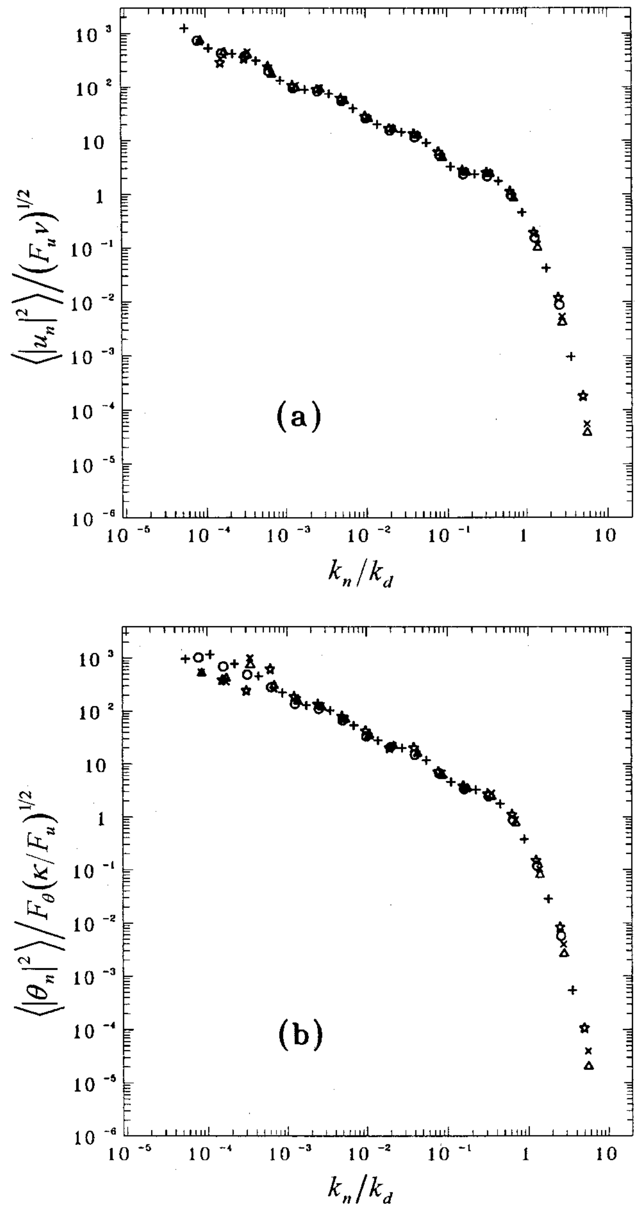

FIG. 3. (a) Velocity spectra and (b) temperature spectra. (The symbols are the same as in Fig. 1.)

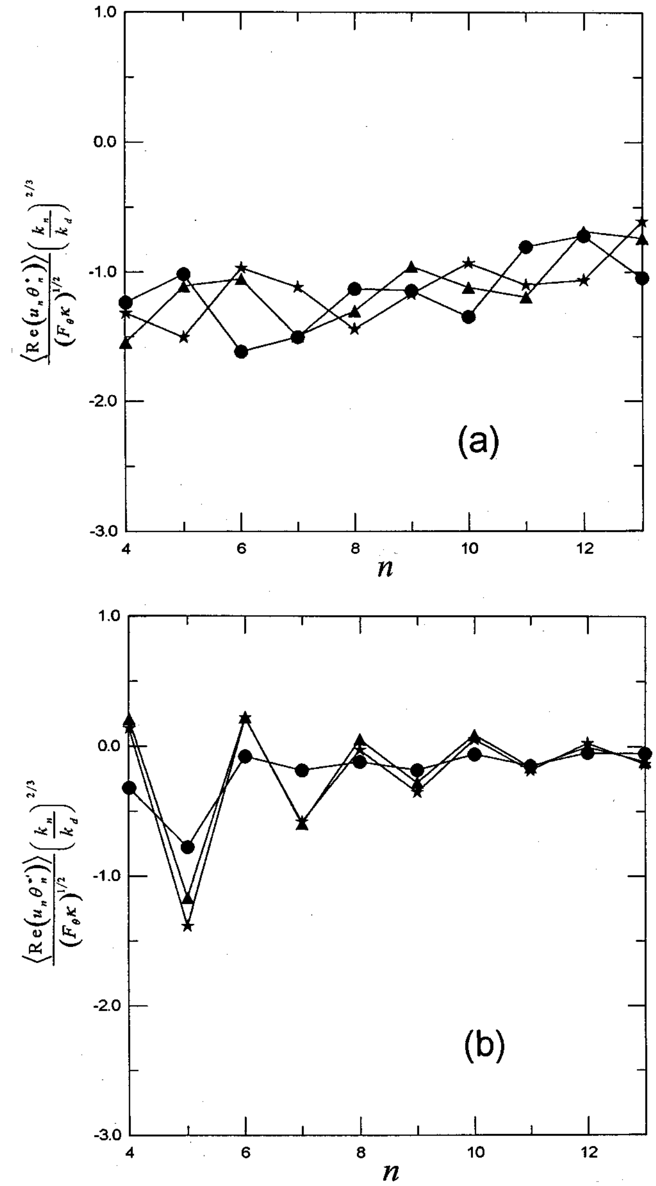

FIG. 4. Normalized exchanging energy $[\langle \mathrm{Re}(u_n \theta_n^*) \rangle / (F_\theta \kappa)^{1/2}](k_n/k_d)^{2/3}$ with $\nu = 10^{-7}$ and $\tau = 0.5$ for (a) model 1 (circles, $\tilde{\alpha} = 0.1$; triangles, $\tilde{\alpha} = 1$; stars, $\tilde{\alpha} = 10$) and (b) model 2 (circles, $\tilde{\alpha} = 0.1$; triangles, $\tilde{\alpha} = 0.2$; stars, $\tilde{\alpha} = 0.3$).

high wave numbers should also decrease slightly. This small correction due to weak buoyancy accounts for the fact that the third-order scaling indices are slightly larger than 1 and they may be sensitive to the changes of $\tilde{\alpha}$, but not to the changes of τ.

However, the above scaling consistency of model 1 does not indicate a full universality by noticing a slight divergence of high-order exponents in Fig. 1. In fact, the probability density functions diversify also in the tails relating to high-flux events. Moreover, the Lyapunov spectra vary with the parameters. Several reasons may account for this. First, because of the strong intermittence of the system, the number of statistical samples may not be large enough to approach a convergent limit. Second, the inertial range may not be wide enough, so that an error always exists when making a least-squares fit. Another important factor is that a systematic error may exist in the model because the EE is conserved only when $\tilde{\alpha} = 0$.

IV. ENERGY SPECTRA

This section discusses the behaviors of normalized spectra and the energy transferring scenario in the model. The energy spectra can be normalized with Kolmogorov scaling. Figure 3(a) shows the velocity spectrum $\langle |u_n|^2 \rangle / (F_u \nu)^{1/2}$ versus the normalized wave number k_n/k_d, where F_u is the ensemble average of the kinetic-energy flux F_u^n within the inertial range and $k_d = (\nu^3/F_u)^{1/4}$. This normalized spectrum has a slope of $\frac{2}{3}$ in the inertial region. Similar universality for the temperature spectra has also been found [Fig. 3(b), where F_θ is the ensemble average of F_ν^n]. Again, while the velocity spectra of model 2 can be normalized by Kolmogorov scaling, the normalized temperature spectra depend strongly on the buoyancy parameter $\tilde{\alpha}$.

In Fig. 3 it is remarkable that the normalized spectra are almost the same forms even in the dissipation range where the power dependence of structure functions does not hold. This implies that under our assumption, some kind of general scaling may be well beyond the range predicted by ESS. Yamada and Ohkitani [20] observed a clear universal scaling of Lyapunov exponents $|\lambda_n|$ (large n) against νk_n^2 in a shell model simulating 2D turbulence. Those exponents of model 1 show similar scaling behaviors. Since the Lyapunov exponent at large n is a large negative number due to the strong dissipation, their consistent scaling is indicative of a universal spectral form in the dissipation range.

It is interesting to show the exchanging energy (Fig. 4). When using model 1, the averaged exchanging energy is negative at any wave number in the inertial range and their absolute values obey Kolmogorov scaling also, that is, the conversion of heat into kinetic energy occurs at each shell [Fig. 4(a)]. To maintain this unidirectional communication, the EE should cascade also accompanying the kinetic energy and entropy cascading. As a matter of fact, we can envisage that in real turbulence the heat is attached to every eddy and transferred to eddies of smaller scales.

However, when applying model 2 the exchanging energy changes signs at different shells [Fig. 4(b)]. This means that under such a coupling mode, the transferring energy between velocity and temperature is not unidirectional shell by shell even in the sense of statistics. Moreover, the absolute values of the exchanging energy show strong oscillatory behaviors when scaled by Kolmogorov scaling. Recalling the scaling laws of structure functions, it seems that the character of the velocity may depend on the nonlinear coupling between velocity components, whereas that of the temperature may depend on both the velocity-velocity and the velocity-scalar coupling. Although the exchanging energy is small compared to the kinetic-energy flux cascaded to small scales, the nonuniformity of it at each shell makes the behavior of temperature totally different. Since the system shows a different energy transfer scenario when changing $\tilde{\alpha}$, model 2 may not be structurally stable.

V. DISCUSSION

From the above results, we see that the conservation laws are crucial for the scaling behaviors of the shell model, perhaps because a conservation quantity may provide a dynamical rule on the energy cascade. From a topological viewpoint, a conservation quantity reduces the motion freedom of the system so that it may behave more consistently and intermittently. In fact, we can design the model based on an artificial conservative quantity $L=\Sigma_{n=0}^{N}k_n^m \mathrm{Re}(u_n^* \vartheta)$ (m is a positive integer) aside from the total energy and entropy. However, when $m\neq 0$, the model depends strongly on the parameters. Therefore, we believe that our EE conservation model may give clues to the universality of a shell model mimicking the thermal convective turbulence.

Our results also suggest a possible EE cascading scenario. While the velocity-velocity coupling provides a chain rule of the kinetic energy of eddies, the velocity-temperature coupling enables the constraint of the heat cascading behaviors. The EE conservation implies that for exchanging energy there is also the same inertial region where the total energy and entropy are cascading without damping. In reality, thermal plumes are randomly released from the boundary layer into the central region, which constantly supplies the necessary heat to drive the motions of large scales [13]. Then, intuitively, the heat may be attached to each eddy and transferred to small eddies when the larger ones break.

It is worth noting that the scaling exponents of the temperature structure functions measured here are obviously different from the experimental results [6], which seems closer to the prediction of Bolgiano-Obukhov (BO) scaling. Brandenburg [8] derived the Bolgiano-Obukhov spectrum for several parameters in his model and found that the result is parameter dependent. Being contrary to the present model, a BO scaling is closely related to the inverse cascading of kinetic energy [8,9]. Procaccia and Zeitak [21] argued that when the Rayleigh number is large enough, a scale range meeting the Bolgiano-Obukhov scaling will be wide enough to be measurable. In contrast, there are some other works [22] in favor of a Kolmogorov spectrum. It is still an unresolved problem in what conditions the model including passive scalars will show BO scaling and behave nearly universally.

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[1] F. Anselmet, Y. Gagne, E. J. Hopfinger, and R. A. Antoma, J. Fluid Mech. **140**, 63 (1984); A. Vincent and M. Meneguzzi, *ibid.* **225**, 1 (1991); C. Meneveau and K. R. Sreenivasan, *ibid.* **224**, 429 (1991).

[2] Z.-S. She and E. Leveque, Phys. Rev. Lett. **72**, 336 (1994).

[3] R. Benzi, S. Ciliberto, R. Tripiccione, C. Baudet, and S. Succi, Phys. Rev. E **48**, R29 (1993).

[4] H. Politano and A. Pouquet, Phys. Rev. E **52**, 636 (1995).

[5] There are many models, e.g., the Obukhov-Corrsin scaling presented by A. M. Obukhov [Izv. Akad. Nauk SSSR. Ser. Geogr. Geofiz. **13**, 58 (1949)] and S. Corrsin [J. Appl. Phys. **22**, 469 (1951)], the log-normal model by A. N. Kolmogorov [J. Fluid Mech. **13**, 82 (1962)] and A. M. Obukhov [*ibid.* **13**, 77 (1962)], and the p model by C. Meneveau and K. R. Sreenivasan [Phys. Rev. Lett. **59**, 1424 (1987).]

[6] R. A. Antonia, E. J. Hopfinger, Y. Gagne, and F. Anselmet, Phys. Rev. A **30**, 2704 (1984).

[7] M. H. Jensen, G. Paladin, and A. Vulpiani, Phys. Rev. A **45**, 7214 (1992); A. Crisanti, M. H. Jensen, G. Paladin, and A. Vulpiani, Physica D **76**, 239 (1994).

[8] A. Brandenburg, Phys. Rev. Lett. **69**, 605 (1992).

[9] E. Suzuki and S. Toh, Phys. Rev. E **51**, 5628 (1995).

[10] P. Frick, B. Dubrulle, and A. Babiano, Phys. Rev. E **51**, 5582 (1995).

[11] L. Kandanoff, D. Lohse, J. Wang, and R. Benzi, Phys. Fluids **7**, 617 (1995).

[12] O. Gat, I. Procaccia, and R. Zeitak, Phys. Rev. E **51**, 1148 (1995).
[13] D. Siggia, Annu. Rev. Fluid Mech. **26**, 137 (1994).
[14] A. S. Monin and A. M. Yaglom, *Statistical Fluid Mechanics* (MIT Press, Cambridge, MA, 1971), Vol. 1.
[15] V. S. L'vov, Phys. Rev. Lett. **67**, 687 (1991); see also S. Toh and E. Suzuki, *ibid.* **73**, 1501 (1994).
[16] C. Basdevant, B. Legras, and R. Sadourny, J. Atmos. Sci. **38**, 2305 (1981).
[17] U. Frisch, Z.-S. She, and O. Thual, J. Fluid Mech. **168**, 221 (1986).
[18] E. Leveque and Z.-S. She, Phys. Rev. Lett. **75**, 2690 (1995).
[19] See, e.g., G. Stolovitzky and K. R. Sreenivasan, Phys. Rev. E **48**, R33 (1993); M. Briscolini, P. Santangelo, S. Succi, and R. Benzi, *ibid.* **50**, R1745 (1994); V. Carbone, P. Veltri, and R. Bruno, Phys. Rev. Lett. **75**, 3110 (1995).
[20] M. Yamada and K. Ohkitani, Phys. Rev. Lett. **60**, 983 (1988).
[21] I. Procaccia and R. Zeitak, Phys. Rev. Lett. **62**, 2128 (1989); Phys. Rev. A **42**, 821 (1990).
[22] B. Castaing, Phys. Rev. Lett. **65**, 3209 (1990); S. Grossman and D. Loshe, *ibid.* **67**, 445 (1991).